9.2 The Pythagorean Theorem

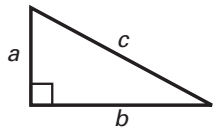
Goals
- Prove the Pythagorean Theorem.
- Use the Pythagorean Theorem to solve problems.

VOCABULARY

Pythagorean triple A Pythagorean triple is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$.

THEOREM 9.4: PYTHAGOREAN THEOREM

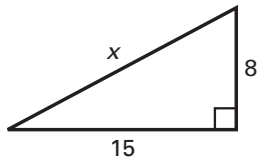
In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



$c^2 = a^2 + b^2$

Example 1 *Finding the Length of a Hypotenuse*

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



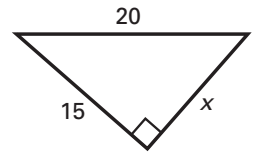
Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$		**Pythagorean Theorem**
$x^2 =$ 8 $^2 +$ 15 ^{2}		**Substitute.**
$x^2 =$ 64 $+$ 225		**Multiply.**
$x^2 =$ 289		**Add.**
$x =$ 17		**Find the positive square root.**

Answer The length of the hypotenuse is 17. Because the side lengths 8, 15, and 17 are integers, they form a Pythagorean triple.

Example 2 *Finding the Length of a Leg*

Find the length of the leg of the right triangle.



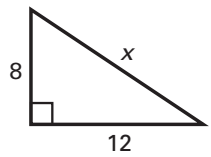
Solution

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$20^2 = x^2 + 15^2$	**Substitute.**
$400 = x^2 + 225$	**Multiply.**
$175 = x^2$	**Subtract 225 from each side.**
$\sqrt{175} = x$	**Find the positive square root.**
$\sqrt{25} \cdot \sqrt{7} = x$	**Use product property.**
$5\sqrt{7} = x$	**Simplify the radical.**

Answer The length of the leg is $5\sqrt{7}$.

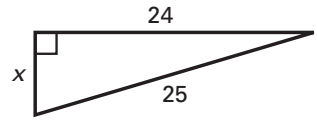
✔ ***Checkpoint*** **Find the value of *x*. Simplify answers that are radicals. Then tell whether the side lengths form a Pythagorean triple.**

1.



$4\sqrt{13}$; No

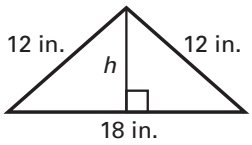
2.



7; Yes

Example 3 ***Finding the Area of a Triangle***

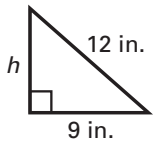
Find the area of the triangle to the nearest tenth of a square inch.



Solution

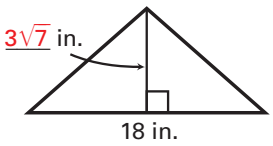
You are given that the base of the triangle is 18 inches, but you do not know the height *h*.

Because the triangle is isosceles, it can be divided into two congruent right angles with the given dimensions. Use the Pythagorean Theorem to find the value of *h*.



$12^2 = 9^2 + h^2$	**Pythagorean Theorem**
$144 = 81 + h^2$	**Multiply.**
$63 = h^2$	**Subtract 81 from each side.**
$\sqrt{63} = h$	**Find the positive square root.**
$3\sqrt{7} = h$	**Simplify the radical.**

Now find the area of the original triangle.

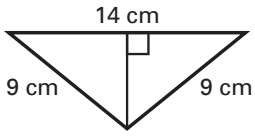


$A = \frac{1}{2}bh$	**Area of a triangle**
$= \frac{1}{2}(18)(3\sqrt{7})$	**Substitute.**
≈ 71.4	**Use a calculator.**

Answer The area of the triangle is about 71.4 square inches.

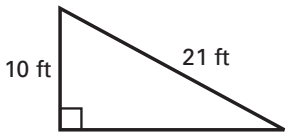
✔ ***Checkpoint*** **Find the area of the triangle. Round your answer to the nearest tenth.**

3.



$39.6\ cm^2$

4.



$92.3\ ft^2$