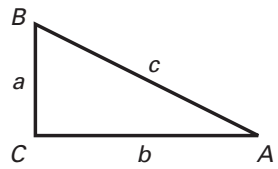
9.3 The Converse of the Pythagorean Theorem

Goals
- Use the Converse of the Pythagorean Theorem to solve problems.
- Use side lengths to classify triangles by their angle measures.

THEOREM 9.5: CONVERSE OF THE PYTHAGOREAN THEOREM

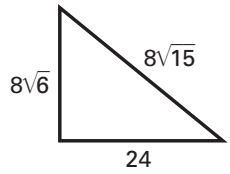
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Example 1 *Verifying Right Triangles*

Tell whether the triangle at the right is a right triangle.



Solution

Let c represent the length of the longest side of the triangle. Check to see whether the side lengths satisfy the equation $c^2 = a^2 + b^2$.

$(8\sqrt{15})^2 \stackrel{?}{=} (8\sqrt{6})^2 + 24^2$

$8^2 \cdot (\sqrt{15})^2 \stackrel{?}{=} 8^2 \cdot (\sqrt{6})^2 + 24^2$

$64 \cdot 15 \stackrel{?}{=} 64 \cdot 6 + 576$

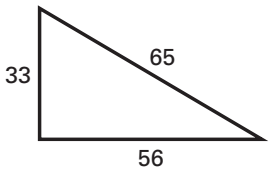
$960 \stackrel{?}{=} 384 + 576$

$960 = 960$

Answer The triangle is a right triangle.

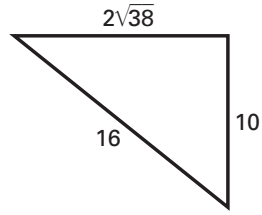
Checkpoint Tell whether the triangle is a right triangle.

1.



The triangle is a right triangle.

2.

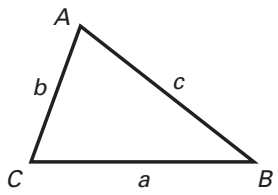


The triangle is not a right triangle.

THEOREM 9.6

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is acute.

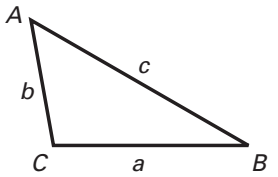
If $c^2 < a^2 + b^2$, then $\triangle ABC$ is acute.



THEOREM 9.7

If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is obtuse.

If $c^2 > a^2 + b^2$, then $\triangle ABC$ is obtuse.



Example 2 ***Classifying Triangles***

Decide whether the set of numbers can represent the side lengths of a triangle. If they can, classify the triangle as *right*, *acute*, or *obtuse*.

a. 28, 40, 48 **b.** 5.7, 12.2, 13.9

Solution

Compare the square of the length of the longest side with the sum of the squares of the lengths of the two shorter sides.

a. c^2 ? $a^2 + b^2$ **Compare c^2 with $a^2 + b^2$.**

48^2 ? 28^2 + 40^2 **Substitute.**

2304 ? 784 + 1600 **Multiply.**

2304 < 2384 **c^2 is** less **than $a^2 + b^2$.**

Answer Because c^2 < $a^2 + b^2$, the triangle is acute.

b. c^2 ? $a^2 + b^2$ **Compare c^2 with $a^2 + b^2$.**

13.9^2 ? 5.7^2 + 12.2^2 **Substitute.**

193.21 ? 32.49 + 148.84 **Multiply.**

193.21 > 181.33 **c^2 is** greater **than $a^2 + b^2$.**

Answer Because c^2 > $a^2 + b^2$, the triangle is obtuse.

✔ ***Checkpoint*** **Can the numbers represent the side lengths of a triangle? If so, classify the triangle as *right*, *acute*, or *obtuse*.**

3. 16, 30, 34 yes; right	**4.** 8, 13, 22 no	**5.** 6, 9, 12 yes; obtuse