9.4 Special Right Triangles

Goals
- Find the side lengths of special right triangles.
- Use special right triangles to solve real-life problems.

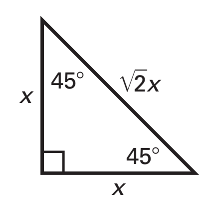
VOCABULARY

Special right triangles Special right triangles are right triangles whose angle measures are 45°-45°-90° or 30°-60°-90°.

THEOREM 9.8: 45°-45°-90° TRIANGLE THEOREM

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

Hypotenuse = $\sqrt{2}$ • leg

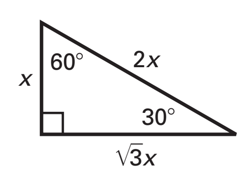


THEOREM 9.9: 30°-60°-90° TRIANGLE THEOREM

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

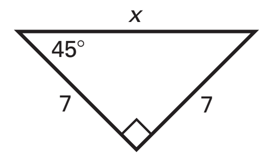
Hypotenuse = 2 • shorter leg

Longer leg = $\sqrt{3}$ • shorter leg



Example 1 ***Finding the Hypotenuse in a 45°-45°-90° Triangle***

Find the value of *x*.



By the Triangle Sum Theorem, the measure of the third angle is 45 °. The triangle is a 45 °- 45 °-90° right triangle, so the length x of the hypotenuse is $\sqrt{2}$ times the length of a leg.

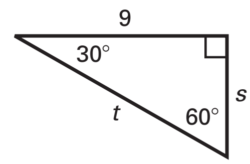
Hypotenuse = $\sqrt{2}$ • leg — **45 °- 45 °-90° Triangle Theorem**

$x = \sqrt{2} \cdot 7$ — **Substitute.**

$x = 7\sqrt{2}$ — **Simplify.**

Example 2 ***Side Lengths in a 30°-60°-90° Triangle***

Find the values of *s* and *t*.



Because the triangle is a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Longer leg = $\sqrt{3}$ • shorter leg — **30°-60°-90° Triangle Theorem**

$9 = \sqrt{3} \cdot s$ — **Substitute.**

$\frac{9}{\sqrt{3}} = s$ — **Divide each side by $\sqrt{3}$.**

$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{9}{\sqrt{3}} = s$ — **Multiply numerator and denominator by $\sqrt{3}$.**

$3\sqrt{3} = s$ — **Simplify.**

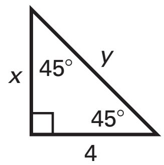
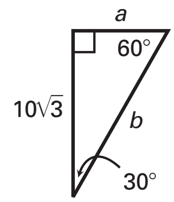
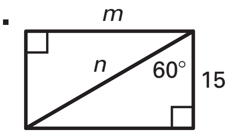
The length of the hypotenuse is twice the length of the shorter leg.

Hypotenuse = 2 • shorter leg — **30°-60°-90° Triangle Theorem**

$t = 2 \cdot 3\sqrt{3}$ — **Substitute.**

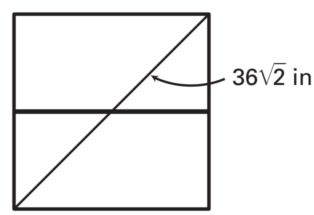
$t = 6\sqrt{3}$ — **Simplify.**

Checkpoint Find the values of the variables.

1.	2.	3.
		
$x = 4$ $y = 4\sqrt{2}$	$a = 10$ $b = 20$	$m = 15\sqrt{3}$ $n = 30$

Example 3 *Finding the Area of a Window*

Construction The window is a square. Find the area of the window.



Solution

First find the side length s of the window by dividing it into two 45°-45°-90° triangles. The length of the hypotenuse is $36\sqrt{2}$ inches. Use this length to find s.

$36\sqrt{2} = \sqrt{2} \cdot s$ **45°-45°-90° Triangle Theorem**

$36 = s$ **Divide each side by $\sqrt{2}$.**

Use $s = 36$ to find the area of the window.

$A = s^2$ **Area of a square**

$= 36^2$ **Substitute.**

$= 1296$ **Multiply.**

Answer The area of the window is 1296 square inches.