9.5 Trigonometric Ratios

Goals
- Find the sine, the cosine, and the tangent of an acute angle.
- Use trigonometric ratios to solve real-life problems.

VOCABULARY

Trigonometric ratio A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

Sine A sine is a trigonometric ratio, abbreviated as sin.

Cosine A cosine is a trigonometric ratio, abbreviated as cos.

Tangent A tangent is a trigonometric ratio, abbreviated as tan.

Angle of elevation An angle of elevation is the angle that your line of sight makes with a horizontal line when you stand and look up at a point in the distance.

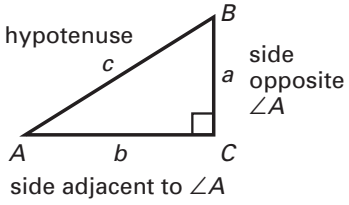
TRIGONOMETRIC RATIOS

Let $\triangle ABC$ be a right triangle. The sine, the cosine, and the tangent of acute $\angle A$ are defined as follows.

$$\sin A = \frac{\text{side opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$$

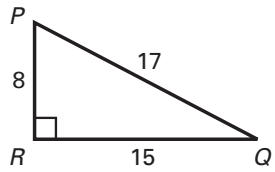
$$\cos A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A} = \frac{a}{b}$$



Example 1 *Finding Trigonometric Ratios*

Find the sine, the cosine, and the tangent of $\angle P$.



Solution

The length of the hypotenuse is 17. The length of the side opposite $\angle P$ is 15, and the length of the side adjacent to $\angle P$ is 8.

$$\sin P = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{17} \approx 0.8824$$

$$\cos P = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{17} \approx 0.4706$$

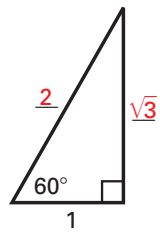
$$\tan P = \frac{\text{opposite}}{\text{adjacent}} = \frac{15}{8} = 1.875$$

Example 2 *Trigonometric Ratios for 60°*

Find the sine, the cosine, and the tangent of 60°.

Solution

Begin by sketching a 30°-60°-90° triangle as shown at the right. To make the calculations simple, choose 1 as the length of the shorter leg. From the 30°-60°-90° Triangle Theorem, it follows that the length of the longer leg is $\sqrt{3}$ and the length of the hypotenuse is 2. Label these lengths in the diagram.

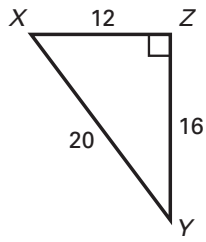


$$\sin 60° = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2} \approx 0.8660$$

$$\cos 60° = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{2} = 0.5$$

$$\tan 60° = \frac{\text{opp.}}{\text{adj.}} = \sqrt{3} \approx 1.7321$$

✔ ***Checkpoint*** **Use the diagram at the right to find the trigonometric ratio.**



1. sin X

$\frac{4}{5} = 0.8$

2. cos X

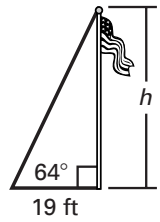
$\frac{3}{5} = 0.6$

3. tan Y

$\frac{3}{4} = 0.75$

Example 3 ***Indirect Measurement***

Flag Pole You are measuring the height of a flag pole. You stand 19 feet from the base of the pole. You measure the angle of elevation from a point on the ground to the top of the pole to be 64°. Estimate the height of the pole.



Solution

$\tan 64° = \frac{\text{opposite}}{\text{adjacent}}$		**Write trigonometric ratio.**
$\tan 64° = \frac{h}{19}$		**Substitute.**
$19 \tan 64° = h$		**Multiply each side by 19.**
$19 (2.0503) = h$		**Evaluate tan 64°.**
$38.9557 \approx h$		**Simplify.**

Answer The height of the flag pole is about 39 feet.