9.6 Solving Right Triangles

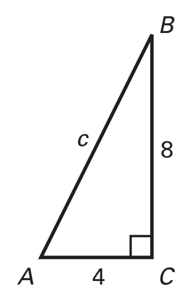
Goals
- Solve a right triangle.
- Use right triangles to solve real-life problems.

VOCABULARY

Solve a right triangle To solve a right triangle means to determine the measures of all three angles and the lengths of all three sides.

Example 1 ***Solving a Right Triangle***

Solve the right triangle. Round decimals to the nearest tenth.



Solution

Use the Pythagorean Theorem to find the length of the hypotenuse c.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$	**Pythagorean Theorem**
$c^2 = \underline{4}^2 + \underline{8}^2$	**Substitute.**
$c^2 = \underline{80}$	**Simplify.**
$c = \underline{4\sqrt{5}}$	**Find the positive square root.**
$c \approx \underline{8.9}$	**Use a calculator to approximate.**

Use a calculator to find the measure of $\angle B$.

($\underline{4}$ ÷ $\underline{8}$) 2nd TAN $\approx \underline{26.6}°$

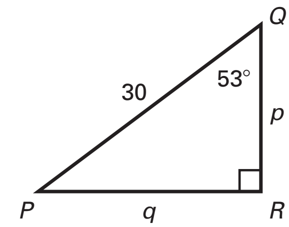
$\angle A$ and $\angle B$ are complementary. The sum of their measures is $\underline{90}°$.

$m\angle A + m\angle B = \underline{90}°$	**$\angle A$ and $\angle B$ are complementary.**
$m\angle A + \underline{26.6}° = \underline{90}°$	**Substitute for $m\angle B$.**
$m\angle A = \underline{63.4}°$	**Subtract.**

Answer The side lengths are $\underline{4}$, $\underline{8}$, and $\underline{8.9}$. The angle measures are $\underline{26.6}°$, $\underline{63.4}°$, and $\underline{90}°$.

Example 2 *Solving a Right Triangle*

Solve the right triangle. Round decimals to the nearest tenth.



Solution

Use trigonometric ratios to find the values of p and q.

$\sin Q = \frac{\text{opp.}}{\text{hyp.}}$ $\cos Q = \frac{\text{adj.}}{\text{hyp.}}$

$\sin \underline{53}° = \frac{q}{\boxed{30}}$ $\cos \underline{53}° = \frac{p}{\boxed{30}}$

$\underline{30} \sin \underline{53}° = q$ $\underline{30} \cos \underline{53}° = p$

$\underline{30}(\underline{0.7986}) \approx q$ $\underline{30}(\underline{0.6018}) \approx p$

$\underline{24.0} \approx q$ $\underline{18.1} \approx p$

$\angle P$ and $\angle Q$ are complementary. The sum of their measures is $\underline{90}°$.

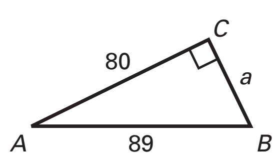
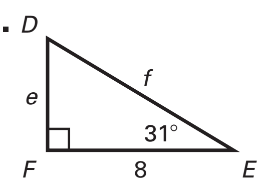
$m\angle P + m\angle Q = \underline{90}°$ **$\angle P$ and $\angle Q$ are complementary.**

$m\angle P + \underline{53}° = \underline{90}°$ **Substitute for $m\angle Q$.**

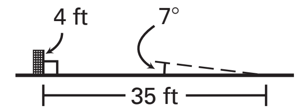
$m\angle P = \underline{37}°$ **Subtract.**

Answer The side lengths of the triangle are 18.1, 24.0, and 30. The angle measures are 37°, 53°, and 90°.

Checkpoint Solve the right triangle. Round decimals to the nearest tenth.

1.	2.
 Side lengths: 39, 80, 89 Angle measures: 26.0°, 64.0°, 90°	 Side lengths: 4.8, 8, 9.3 Angle measures: 31°, 59°, 90°

Example 3 *Solving a Right Triangle*



Sports When a hockey player is 35 feet from the goal line, he shoots the puck directly at the goal. The angle of elevation at which the puck leaves the ice is 7°. The height of the goal is 4 feet. Will the player score a goal?

Solution

Begin by finding the height h of the puck at the goal line. Use a trigonometric ratio.

$\tan \underline{7}° = \frac{\text{opp.}}{\text{adj.}}$ **Write trigonometric ratio.**

$\tan \underline{7}° = \frac{h}{\boxed{35}}$ **Substitute.**

$\underline{35} \tan \underline{7}° = h$ **Multiply each side by $\underline{35}$.**

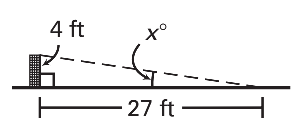
$\underline{35}(\underline{0.1228}) \approx h$ **Use a calculator.**

$\underline{4.3} \approx h$ **Multiply.**

Answer Because the height of the puck at the goal line (_4.3_ feet) is greater than the height of the goal (4 feet), the player _will not_ score a goal.

✔ *Checkpoint* **Complete the following exercise.**

3. A hockey player is 27 feet from the goal line. He shoots the puck directly at the goal. The height of the goal is 4 feet. What is the maximum angle of elevation at which the player can shoot the puck and still score a goal?



8.4°