9.7 Vectors

Goals • Find the magnitude and direction of a vector.
• Add vectors.

VOCABULARY

Magnitude of a vector The magnitude of a vector is the distance from the initial point to the terminal point.

Direction of a vector The direction of a vector is determined by the angle that the vector makes with a horizontal line.

Equal vectors Two vectors are equal when they have the same magnitude and direction.

Parallel vectors Two vectors are parallel when they have the same or opposite direction.

Sum of vectors The sum of two vectors is a vector that joins the initial point of the first vector and the terminal point of the second vector.

Example 1 ***Finding the Magnitude of a Vector***

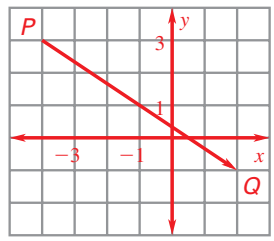
$P(-4, 3)$ and $Q(2, -1)$ are the initial and terminal points of a vector. Draw $\overrightarrow{PQ}$ in a coordinate plane. Then find its magnitude.

Solution

Component form $= \langle x_2 - x_1, y_2 - y_1 \rangle$

$\overrightarrow{PQ} = \langle \underline{2} - \underline{(-4)}, \underline{-1} - \underline{3} \rangle$

$= \langle \underline{6}, \underline{-4} \rangle$

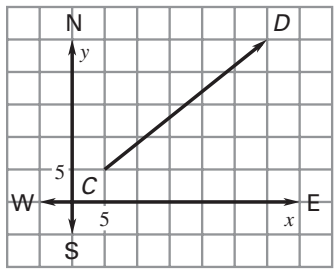


Use the Distance Formula to find the magnitude.

$$|\overrightarrow{PQ}| = \sqrt{[\underline{2} - (\underline{-4})]^2 + (\underline{-1} - \underline{3})^2} = \sqrt{\underline{52}} \approx \underline{7.2}$$

Example 2 ***Describing the Direction of a Vector***

The vector $\overrightarrow{CD}$ describes the velocity of a moving hot air balloon. The scale on each axis is in miles per hour.



a. Find the speed of the balloon.

b. Find the direction it is traveling relative to east.

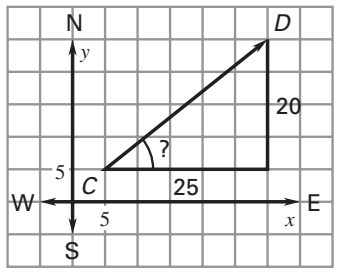
Solution

a. The magnitude of the vector $\overrightarrow{CD}$ represents the balloon's speed. Use the Distance Formula.

$$|\overrightarrow{CD}| = \underline{\sqrt{(30-5)^2+(25-5)^2}} = \underline{\sqrt{1025}} \approx \underline{32.0}$$

Answer The speed of the balloon is about 32 miles per hour.

b. The tangent of the angle formed by the vector and a line drawn parallel to the x-axis is $\frac{20}{25}$, or 0.8. Use a calculator to find the angle measure.

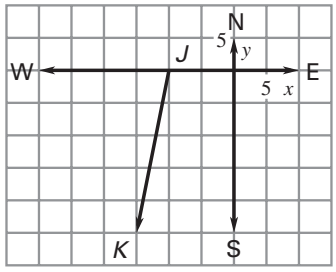


0.8 [2nd] [TAN] $\approx$ 38.7°

Answer The balloon is traveling in a direction about 38.7° north of east.

✔ ***Checkpoint*** **Complete the following exercise.**

1. The vector represents the velocity of a moving hot air balloon. The scale on each axis is in miles per hour. Find the balloon's speed and direction relative to west.



25.5 miles per hour; 78.7° south of west

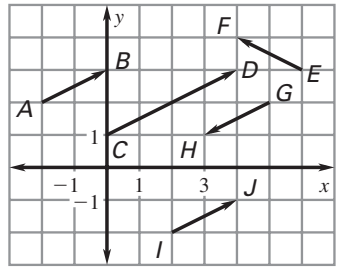
Example 3 *Identifying Equal and Parallel Vectors*

In the diagram, these vectors have the same direction: $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{IJ}$.

These vectors are equal: $\overrightarrow{AB}$, $\overrightarrow{IJ}$.

These vectors are parallel:

$\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{GH}$, $\overrightarrow{IJ}$.



ADDING VECTORS

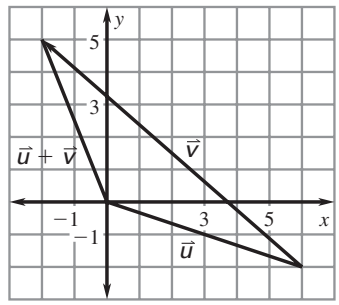
Sum of Two Vectors

The sum of $\vec{u} = \langle a_1, b_1 \rangle$ and $\vec{v} = \langle a_2, b_2 \rangle$ is

$\vec{u} + \vec{v} = \langle a_1 + a_2, b_1 + b_2 \rangle$.

Example 4 *Finding the Sum of Two Vectors*

Let $\vec{u} = \langle 6, -2 \rangle$ and $\vec{v} = \langle -8, 7 \rangle$. Find the sum $\vec{u} + \vec{v}$.



Solution

To find the sum, add the horizontal components and add the vertical components of u and v.

$\vec{u} + \vec{v} = \langle 6 + (-8), -2 + 7 \rangle$

$= \langle -2, 5 \rangle$

✔ ***Checkpoint*** **Find the sum of the vectors.**

2. $\langle 6, 0 \rangle, \langle 1, 3 \rangle$ $\langle 7, 3 \rangle$	**3.** $\langle -5, 2 \rangle, \langle 7, -6 \rangle$ $\langle 2, -4 \rangle$